



Variable-tension lightlike brane as a gravitational source of traversable Misner–Wheeler-type wormholes

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ABSTRACT

Consistent Lagrangian description of variable-tension lightlike p -branes (LL -branes) is presented in two equivalent forms – a Polyakov-type formulation and a dual to it Nambu–Goto-type formulation. An important and non-standard characteristic feature of the LL -branes is that the brane tension appears as a non-trivial additional dynamical degree of freedom. We consider properties of $p = 2$ LL -brane dynamics (as a test brane) in $D = 4$ Kerr or Kerr–Newman gravitational backgrounds in some detail. It is shown that the LL -brane automatically positions itself on the horizon and rotates along with the same angular velocity.

Finally, we construct explicitly a traversable wormhole of Misner–Wheeler type based on a Reissner–Nordström geometry. This wormhole is constructed as a self-consistent solution of the electrically sourceless Einstein–Maxwell system in the $D = 4$ bulk interacting with a LL -brane. The pertinent wormhole throat is located precisely at the LL -brane sitting on the outer Reissner–Nordström horizon with the Reissner–Nordström mass and charge being functions of the dynamical LL -brane tension.

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1. Introduction

Lightlike branes (LL -branes, for short) attract special interest in general relativity. This is due primarily because of their role in the effective description of many cosmological and astrophysical effects: (a) impulsive lightlike signals arising in cataclysmic astrophysical events [1]; (b) the “membrane paradigm” theory of black hole physics [2]; (c) thin-wall approach to domain walls coupled to gravity [3,4]. More recently LL -branes acquired significance also in the context of modern non-perturbative string theory [5].

Our formalism makes an essential use of an alternative non-Riemannian measure of integration (volume-form). The latter leads to different type of gravitational theories [6] which address various basic problems of cosmological interest. In the context of the theory of extended objects employing an alternative integration measure independent of the intrinsic Riemannian metric on the world-volume within the Polyakov-type approach leads to a dynamical string/brane tension [7]. Furthermore it allows the con-

struction of consistent Lagrangian actions describing intrinsically lightlike p -branes (LL -branes) [8]. Also an equivalent Nambu–Goto-type formulation of LL -brane dynamics has been shown to exist (third Ref. [9], cf. Eq. (17) below) which is dual to the Polyakov-type formulation (cf. Eq. (1) below).

The above mentioned basic properties of LL -branes (intrinsically lightlike brane modes and variable dynamical tension) are in sharp contrast w.r.t. those of ordinary Nambu–Goto branes, which describe massive modes and where the brane tension is given as an *ad hoc* constant.

In a series of papers [8,9] we have studied the properties of LL -branes both as test branes moving in physically interesting gravitational backgrounds, as well as material and charge sources for gravity and electromagnetism in self-consistent bulk gravity-matter systems interacting with LL -branes.

In gravitational backgrounds of spherically symmetric type and codimension-one a general feature of LL -branes is that their dynamics is consistent only provided the background possesses an event horizon which is automatically occupied by the LL -brane. Also, the dynamical brane tension exhibits an exponential “inflation/deflation” property analogous to the “mass inflation effect” around black hole horizons discovered in [10]. Furthermore, unlike conventional braneworlds, where the underlying branes are of Nambu–Goto type and in their ground state they position themselves at some fixed point in the extra dimensions of the bulk

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space–time, codimension-two (or more) *lightlike braneworlds* perform in the ground state non-trivial motions in the extra dimensions – planar circular, spiral winding *etc* depending on the topology of the extra dimensions. For details we refer to [8,9].

In the present Letter we are going to study dynamics of *LL-branes* both as test branes and material sources in the case of Kerr–Newman black hole space–time, in particular – Reissner–Nordström space–time as a limiting case of the former (cf. the textbooks [11]). We find that the *LL-brane* automatically positions itself on the Kerr–Newman horizon and in addition it rotates along with the same angular velocity as the black hole. When moving as a test brane in Kerr–Newman background we find exponential “inflation/deflation” of the dynamical *LL-branes* tension similar to the spherically symmetric case.

Our next task is to construct a traversable wormhole solution of Kerr–Newman type of the self-consistent Einstein–Maxwell system interacting with a *LL-brane*, *i.e.*, invoking the *LL-brane* as a material source of the wormhole. It turns out that our construction works only for the spherically symmetric limiting case of Reissner–Nordström geometry. It is achieved by “surgically” eliminating the space–time region inside the (outer) Reissner–Nordström horizon and sewing together two copies of the exterior Reissner–Nordström region along their common (outer) horizon through the *LL-brane*’s energy–momentum tensor (derived from the pertinent *LL-brane* world-volume action). In other words, one achieves a Reissner–Nordström traversable wormhole solution of Misner–Wheeler type [12] which does not require electrically charged sources.²

Let us particularly stress that our construction below is based on a first principle’s approach, *i.e.*, the “surgical” matching at the Reissner–Nordström wormhole “throat” comes from a well-defined world-volume Lagrangian description of *LL-brane* dynamics.

2. World-volume Lagrangian description of lightlike branes

In Refs. [8,9] we have proposed a systematic Lagrangian formulation of a generalized Polyakov-type for *LL-branes* in terms of the world-volume action:

$$S_{LL} = \int d^{p+1} \sigma \Phi(\varphi) \left[-\frac{1}{2} \gamma^{ab} g_{ab} + L(F^2) \right], \quad (1)$$

with the following notations. Here γ_{ab} denotes the intrinsic Riemannian metric on the world-volume, $a, b = 0, 1, \dots, p$; $(\sigma^a) \equiv (\tau, \sigma^i)$ with $i = 1, \dots, p$; g_{ab} is the induced metric:

$$g_{ab} \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X), \quad (2)$$

which becomes *singular* on-shell (manifestation of the lightlike nature, cf. Eq. (8) below);

$$\Phi(\varphi) \equiv \frac{1}{(p+1)!} \varepsilon_{I_1 \dots I_{p+1}} \varepsilon^{a_1 \dots a_{p+1}} \partial_{a_1} \varphi^{I_1} \dots \partial_{a_{p+1}} \varphi^{I_{p+1}} \quad (3)$$

is an alternative non-Riemannian reparametrization-covariant integration measure density replacing the standard $\sqrt{-\gamma} \equiv \sqrt{-\det \|\gamma_{ab}\|}$ and built from auxiliary world-volume scalars $\{\varphi^I\}_{I=1}^{p+1}$;

$$F^2 \equiv F_{a_1 \dots a_p} F_{b_1 \dots b_p} \gamma^{a_1 b_1} \dots \gamma^{a_p b_p},$$

where:

$$F_{a_1 \dots a_p} = p \partial_{[a_1} A_{a_2 \dots a_p]}, \quad F^{*a} = \frac{1}{p!} \frac{\varepsilon^{aa_1 \dots a_p}}{\sqrt{-\gamma}} F_{a_1 \dots a_p} \quad (4)$$

are the field-strength and its dual one of an auxiliary world-volume $(p-1)$ -rank antisymmetric tensor gauge field $A_{a_1 \dots a_{p-1}}$ with Lagrangian $L(F^2)$.

Equivalently one can rewrite (1) as:

$$S_{LL} = \int d^{p+1} \sigma \chi \sqrt{-\gamma} \left[-\frac{1}{2} \gamma^{ab} g_{ab} + L(F^2) \right], \quad \chi \equiv \frac{\Phi(\varphi)}{\sqrt{-\gamma}}, \quad (5)$$

where from we see that the composite field χ plays the role of a *dynamical (variable) brane tension*.

Remark 1. For the special choice $L(F^2) = (F^2)^{1/p}$ the above action becomes invariant under Weyl (conformal) symmetry:

$$\gamma_{ab} \longrightarrow \gamma'_{ab} = \rho \gamma_{ab}, \quad \varphi^i \longrightarrow \varphi'^i = \varphi^i(\varphi) \quad (6)$$

with Jacobian $\det \|\frac{\partial \varphi'^i}{\partial \varphi^j}\| = \rho$.

Now let us consider the equations of motion corresponding to (1) w.r.t. φ^I :

$$\partial_a \left[\frac{1}{2} \gamma^{cd} g_{cd} - L(F^2) \right] = 0 \longrightarrow \frac{1}{2} \gamma^{cd} g_{cd} - L(F^2) = M, \quad (7)$$

where M is an arbitrary integration constant. The equations of motion w.r.t. γ^{ab} read:

$$\frac{1}{2} g_{ab} - F^2 L'(F^2) \left[\gamma_{ab} - \frac{F_a^* F_b^*}{F^{*2}} \right] = 0, \quad (8)$$

where F^{*a} is the dual field strength (4).

There are two important consequences of Eqs. (7), (8). First, both of them taken together imply the constraint:

$$L(F^2) - p F^2 L'(F^2) + M = 0, \quad (9)$$

i.e. $F^2 = F^2(M) = \text{const}$ on-shell.

Second, Eq. (8) exhibits *on-shell singularity* of the induced metric (2):

$$g_{ab} F^{*b} \equiv \partial_a X^\mu G_{\mu\nu} (F^{*b} \partial_b X^\nu) = 0, \quad (10)$$

i.e., the tangent vector to the world-volume $F^{*a} \partial_a X^\mu$ is lightlike w.r.t. metric of the embedding space–time.

Remark 2. Let us stress the importance of introducing the alternative non-Riemannian integration measure density in the form (3). If we would have started with world-volume *LL-brane* action in the form (5) where the tension χ is an *elementary* field (instead of being function of the measure-density scalars), then variation w.r.t. χ would produce second Eq. (7) with M identically zero. This in turn by virtue of the constraint (9) (with $M = 0$) would require the Lagrangian $L(F^2)$ to assume the special fractional function form from Remark 1 above. This special case of Weyl-conformally invariant *LL-branes* has been discussed in our previous papers (first two Refs. [8]).

Further, the equations of motion w.r.t. world-volume gauge field $A_{a_1 \dots a_{p-1}}$ (with χ as defined in (5) and accounting for the constraint (9)) read:

$$\partial_{[a} (F_{b]}^* \chi) = 0. \quad (11)$$

They allow us to introduce the dual “gauge” potential u :

$$F_a^* = \text{const} \cdot \frac{1}{\chi} \partial_a u, \quad (12)$$

enabling us to rewrite Eq. (8) (the lightlike constraint) in terms of the dual potential u in the form:

² Misner and Wheeler [12] discovered that wormholes connecting two asymptotically flat spacetimes provide the possibility of “charge without charge”, *i.e.*, electromagnetically non-trivial solutions where the lines of force of the electric field flow from one universe to the other without a source and giving the impression of being positively charged in one universe and negatively charged in the other universe.

$$\gamma_{ab} = \frac{1}{2a_0} g_{ab} - \frac{2}{\chi^2} \partial_a u \partial_b u, \quad a_0 \equiv F^2 L'(F^2)|_{F^2=F^2(M)} = \text{const} \quad (13)$$

($L'(F^2)$ denotes derivative of $L(F^2)$ w.r.t. the argument F^2). From (12) and (9) we obtain the relation:

$$\chi^2 = -2\gamma^{ab} \partial_a u \partial_b u, \quad (14)$$

and the Bianchi identity $\nabla_a F^{*a} = 0$ becomes:

$$\partial_a \left(\frac{1}{\chi} \sqrt{-\gamma} \gamma^{ab} \partial_b u \right) = 0. \quad (15)$$

Finally, the X^μ equations of motion produced by the (1) read:

$$\partial_a (\chi \sqrt{-\gamma} \gamma^{ab} \partial_b X^\mu) + \chi \sqrt{-\gamma} \gamma^{ab} \partial_a X^\nu \partial_b X^\lambda \Gamma_{\nu\lambda}^\mu(X) = 0 \quad (16)$$

where $\Gamma_{\nu\lambda}^\mu = \frac{1}{2} G^{\mu\kappa} (\partial_\nu G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\kappa G_{\nu\lambda})$ is the Christoffel connection for the external metric.

Now it is straightforward to prove that the system of equations (14)–(16) for (X^μ, u, χ) , which are equivalent to the equations of motion (7)–(11), (16) resulting from the original Polyakov-type *LL-brane* action (1), can be equivalently derived from the following *dual* Nambu–Goto-type world-volume action:

$$S_{\text{NG}} = - \int d^{p+1} \sigma T \sqrt{-\det \left\| g_{ab} - \frac{1}{T^2} \partial_a u \partial_b u \right\|}. \quad (17)$$

Here g_{ab} is the induced metric (2), T is *dynamical* tension simply related to the dynamical tension χ from the Polyakov-type formulation (5) as $T^2 = \frac{\chi^2}{4a_0}$ with a_0 – same constant as in (13).

In what follows we will consider the initial Polyakov-type form (1) of the *LL-brane* world-volume action. World-volume reparametrization invariance allows to introduce the standard synchronous gauge-fixing conditions:

$$\gamma^{0i} = 0 \quad (i = 1, \dots, p), \quad \gamma^{00} = -1. \quad (18)$$

Also, we will use a natural ansatz for the “electric” part of the auxiliary world-volume gauge field-strength:

$$F^{*i} = 0 \quad (i = 1, \dots, p), \quad \text{i.e. } F_{0i_1 \dots i_{p-1}} = 0, \quad (19)$$

meaning that we choose the lightlike direction in Eq. (10) to coincide with the brane proper-time direction on the world-volume ($F^{*a} \partial_a \simeq \partial_\tau$). The Bianchi identity ($\nabla_a F^{*a} = 0$) together with (18)–(19) and the definition for the dual field-strength in (4) imply:

$$\partial_0 \gamma^{(p)} = 0 \quad \text{where } \gamma^{(p)} \equiv \det \|\gamma_{ij}\|. \quad (20)$$

Then *LL-brane* equations of motion acquire the form (recall definition of g_{ab} (2)):

$$g_{00} \equiv \dot{X}^\mu G_{\mu\nu} \dot{X}^\nu = 0, \quad g_{0i} = 0, \quad g_{ij} - 2a_0 \gamma_{ij} = 0 \quad (21)$$

(the latter are analogs of Virasoro constraints), where the M -dependent constant a_0 (the same as in (13)) must be strictly positive:

$$\partial_i \chi = 0 \quad (\text{remnant of Eq. (11)}), \quad (22)$$

$$-\sqrt{\gamma^{(p)}} \partial_0 (\chi \partial_0 X^\mu) + \partial_i \left(\chi \sqrt{\gamma^{(p)}} \gamma^{ij} \partial_j X^\mu \right) + \chi \sqrt{\gamma^{(p)}} (-\partial_0 X^\nu \partial_0 X^\lambda + \gamma^{kl} \partial_k X^\nu \partial_l X^\lambda) \Gamma_{\nu\lambda}^\mu = 0. \quad (23)$$

3. Lightlike branes in Kerr–Newman black hole background

Let us consider ($D = 4$)-dimensional Kerr–Newman background metric in the standard Boyer–Lindquist coordinates (see e.g. [11]):

$$ds^2 = -A(dt)^2 - 2E dt d\varphi + \frac{\Sigma}{\Delta} (dr)^2 + \Sigma(d\theta)^2 + D \sin^2 \theta (d\varphi)^2, \quad (24)$$

$$A \equiv \frac{\Delta - a^2 \sin^2 \theta}{\Sigma}, \quad E \equiv \frac{a \sin^2 \theta (r^2 + a^2 - \Delta)}{\Sigma}, \quad (25)$$

$$D \equiv \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta}{\Sigma},$$

where $\Sigma \equiv r^2 + a^2 \cos^2 \theta$, $\Delta \equiv r^2 + a^2 + e^2 - 2mr$. Let us recall that the Kerr–Newman metric (24)–(25) reduces to the Reissner–Nordström metric in the limiting case $a = 0$.

For the *LL-brane* embedding we will use the following ansatz:

$$X^0 \equiv t = \tau, \quad r = r(\tau), \quad \theta = \sigma^1, \quad \varphi = \sigma^2 + \tilde{\varphi}(\tau). \quad (26)$$

In this case the *LL-brane* equations of motion (20)–(21) acquire the form:

$$-A + \frac{\Sigma}{\Delta} \dot{r}^2 + D \sin^2 \theta \dot{\varphi}^2 - 2E \dot{\varphi} = 0, \quad (27)$$

$$-E + D \sin^2 \theta \dot{\varphi} = 0, \quad \frac{d}{d\tau} (D \Sigma \sin^2 \theta) = 0.$$

Inserting the ansatz (26) into (27) the last Eq. (27) implies:

$$r(\tau) = r_0 = \text{const}, \quad (28)$$

whereas the second Eq. (27) yields:

$$\Delta(r_0) = 0, \quad \omega \equiv \dot{\varphi} = \frac{a}{r_0^2 + a^2}. \quad (29)$$

Eqs. (28)–(29) indicate that:

- (i) The *LL-brane* automatically locates itself on the Kerr–Newman horizon $r = r_0$ – horizon “straddling” according to the terminology of the first Ref. [4];
- (ii) The *LL-brane* rotates along with the same angular velocity ω as the Kerr–Newman horizon.

The first Eq. (27) implies that \dot{r} vanishes on-shell as:

$$\dot{r} \simeq \pm \frac{\Delta(r)}{r_0^2 + a^2} \Big|_{r \rightarrow r_0}. \quad (30)$$

We will also need the explicit form of the last Eq. (21) (using notations (25)):

$$\gamma_{ij} = \frac{1}{2a_0} \begin{pmatrix} \Sigma & 0 \\ 0 & D \sin^2 \theta \end{pmatrix} \Big|_{r=r_0, \theta=\sigma^1}. \quad (31)$$

Among the X^μ -equations of motion (23) only the X^0 -equation yields additional information. Because of the embedding $X^0 = \tau$ it acquires the form of a time-evolution equation for the dynamical brane tension χ :

$$\partial_\tau \chi + \chi [\partial_\tau X^\nu \partial_\tau X^\lambda - \gamma^{ij} \partial_i X^\nu \partial_j X^\lambda] \Gamma_{\nu\lambda}^0 = 0, \quad (32)$$

which, after taking into account (26), (28)–(29) and the explicit expressions for the Kerr–Newman Christoffel connection coefficients (first Ref. [11]), reduces to:

$$\partial_\tau \chi + \chi 2\dot{r} \left[\Gamma_{0r}^0 + \frac{a}{r_0^2 + a^2} \Gamma_{r\varphi}^0 \right]_{r=r_0} = 0. \quad (33)$$

Singularity on the horizon of the Christoffel coefficients ($\simeq \Delta^{-1}$) appearing in (33) is cancelled by Δ in \dot{r} (30) so that finally we obtain:

$$\partial_\tau \chi \pm \chi \frac{2(r_0 - m)}{r_0^2 + a^2} = 0, \quad \text{i.e. } \chi = \chi_0 \exp \left\{ \mp 2 \frac{(r_0 - m)}{r_0^2 + a^2} \tau \right\}. \quad (34)$$

Thus, we find “mass inflation/deflation” effect (according to the terminology of [10]) on the Kerr–Newman horizon via the exponential time dependence of the dynamical *LL-brane* tension. The latter is an analog of the previously found “mass inflation/deflation” effect with *LL-branes* in spherically symmetric gravitational backgrounds [9].

4. Bulk Einstein–Maxwell system interacting with a lightlike brane and Misner–Wheeler traversable wormhole solution

Now we will consider a self-consistent $D = 4$ Einstein–Maxwell system free of electrically charged matter, coupled to a LL -brane where the LL -brane will serve as a gravitational source through its energy–momentum tensor:

$$S = \int d^4x \sqrt{-G} \left[\frac{R(G)}{16\pi} - \frac{1}{4} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} \right] + S_{LL}. \quad (35)$$

Here $\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu$ and S_{LL} is the same LL -brane world-volume action as in (5). The pertinent Einstein–Maxwell equations of motion read:

$$R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R = 8\pi (T_{\mu\nu}^{(EM)} + T_{\mu\nu}^{(brane)}), \quad (36)$$

$$\partial_\nu (\sqrt{-G} G^{\mu\kappa} G^{\nu\lambda} \mathcal{F}_{\kappa\lambda}) = 0,$$

where $T_{\mu\nu}^{(EM)} = \mathcal{F}_{\mu\kappa} \mathcal{F}_{\nu\lambda} G^{\kappa\lambda} - G_{\mu\nu} \frac{1}{4} \mathcal{F}_{\rho\kappa} \mathcal{F}_{\sigma\lambda} G^{\rho\sigma} G^{\kappa\lambda}$, and the LL -brane energy–momentum tensor is straightforwardly derived from (5):

$$T_{\mu\nu}^{(brane)} = -G_{\mu\kappa} G_{\nu\lambda} \int d^3\sigma \frac{\delta^{(4)}(x - X(\sigma))}{\sqrt{-G}} \chi \sqrt{-\gamma} \gamma^{ab} \partial_a X^\kappa \partial_b X^\lambda. \quad (37)$$

Following the standard procedure [13] we will now construct a traversable wormhole solution to the Einstein–Maxwell equations (36) of Misner–Wheeler type [12] by making an essential use of the explicit expression for the LL -brane energy–momentum tensor (37). Namely, let us take two copies of Kerr–Newman exterior space–time region, *i.e.*, solutions to (36) with $G_{\mu\nu}$ as in (24)–(25) for $r > r_0$, where $r_0 = m + \sqrt{m^2 - a^2 - e^2}$ is the outer horizon radius, and let us try to sew the two regions together along the horizon $r = r_0$ via the LL -brane. To this end it is customary to introduce a new radial-like coordinate η normal w.r.t. the LL -brane:

$$r = r_0 + |\eta|, \quad \frac{\partial r}{\partial \eta} = \text{sign}(\eta), \quad \eta \in (-\infty, +\infty). \quad (38)$$

Accordingly, the metric in the total space of the two copies of exterior Kerr–Newman regions reads:

$$ds^2 = -\tilde{A}(dt)^2 - 2\tilde{E} dt d\varphi + \frac{\tilde{\Sigma}}{\tilde{\Delta}} (d\eta)^2 + \tilde{\Sigma}(d\theta)^2 + \tilde{D} \sin^2 \theta (d\varphi)^2, \quad (39)$$

where $\tilde{A} \equiv A|_{r=r_0+|\eta|}$ with the same A as in (24)–(25), and similarly for \tilde{E} , $\tilde{\Sigma}$, $\tilde{\Delta}$, \tilde{D} . The two copies transform into each other under the “parity” transformation $\eta \rightarrow -\eta$.

Inserting in (37) the expressions for $X^\mu(\sigma)$ from (26) and (28)–(29), taking into account the explicit form of Kerr–Newman metric coefficients (24)–(25), the gauge-fixing conditions for the intrinsic world-volume metric (18) and Eq. (31) we get:

$$T_{(brane)}^{\mu\nu} = S^{\mu\nu} \delta(\eta) \quad (40)$$

with surface energy–momentum tensor:

$$S^{\mu\nu} \equiv -\frac{\chi}{2a_0} \frac{r_0^2 + a^2}{r_0^2 + a^2 \cos^2 \theta} \times \left[-\partial_\tau X^\mu \partial_\tau X^\nu + \gamma^{ij} \partial_i X^\mu \partial_j X^\nu \right]_{t=\tau, \theta=\sigma^1, \varphi=\sigma^2 + \frac{a}{r_0^2+a^2} \tau}, \quad (41)$$

where now the indices μ, ν refer to $(t, \eta, \theta, \varphi)$ and a_0 is the integration constant parameter appearing in the LL -brane dynamics (cf. Eq. (13)). Let us also note that unlike the case of test LL -brane moving in a Kerr–Newman background (Eqs. (32)–(34)), the dynamical tension χ in Eq. (41) is *constant*. This is due to the fact that in

the present context we have a discontinuity in the Kerr–Newman Christoffel connection coefficients across the LL -brane sitting on the horizon ($\eta = 0$). The latter problem in treating the geodesic LL -brane equations of motion (16), in particular – Eq. (32), is resolved following the approach in Ref. [3] (see also the regularization approach in Ref. [14], Appendix A) by taking the mean value of the pertinent non-zero Christoffel coefficients across the discontinuity at $\eta = 0$ and accounting for (38):

$$\langle \Gamma_{0\eta}^0 \rangle \equiv \frac{1}{2} (\Gamma_{\partial\eta}^0|_{\eta \rightarrow +0} + \Gamma_{\partial\eta}^0|_{\eta \rightarrow -0}) = \frac{1}{2} (\Gamma_{0r}^0|_{r \rightarrow r_0} - \Gamma_{0r}^0|_{r \rightarrow r_0}) = 0, \quad (42)$$

and similarly for $\langle \Gamma_{\eta\varphi}^0 \rangle = 0$. Therefore, in the latter case Eq. (32) is reduced to $\partial_\tau \chi = 0$.

From the Einstein equations (36), taking into account Eqs. (40)–(41), one obtains in a standard way the discontinuity for the Kerr–Newman Christoffel coefficients (analog of Israel junction conditions [3,4]). Namely, observing that:

$$R_{\mu\nu} \equiv \partial_\eta \Gamma_{\mu\nu}^\eta + \partial_\mu \partial_\nu \ln \sqrt{-G} + \text{non-singular terms} = 8\pi \left(S_{\mu\nu} - \frac{1}{2} G_{\mu\nu} S^\lambda{}_\lambda \right) \delta(\eta) + \text{non-singular terms}, \quad (43)$$

we find that delta-function singularities are present on both sides for $(\mu\nu) = (\eta\eta)$. For $(\mu\nu) = (0\eta)$ and $(\mu\nu) = (\eta\varphi)$ such singularities appear only on the r.h.s., and the rest of (43) are singularity free. Consistency of (43) for $(\mu\nu) = (0\eta)$ and $(\mu\nu) = (\eta\varphi)$, *i.e.*, vanishing of the delta-function singularity on the r.h.s. requires $a = 0$. In other words, consistent wormhole solution with LL -brane as a “throat” may exist only for the limiting case of spherically symmetric Reissner–Nordström geometry.

It remains to check Eq. (43) for $(\mu\nu) = (\eta\eta)$. In order to avoid coordinate singularity on the horizon it is more convenient to consider the mixed component version of the latter:

$$R_{\eta}^\eta = 8\pi \left(S_{\eta}^\eta - \frac{1}{2} S^\lambda{}_\lambda \right) \delta(\eta) + \text{non-singular terms}. \quad (44)$$

Evaluating the r.h.s. of (44) from (41) with (26) and (28)–(29) we obtain:

$$\partial_\eta ((r_0 + |\eta|)^2 \partial_\eta \tilde{A}) = -16\pi r_0^2 \chi \delta(\eta) + \text{non-singular terms}, \quad (45)$$

where $\tilde{A} = (1 - \frac{2m}{r} + \frac{e^2}{r^2})_{r=r_0+|\eta|}$ is the Reissner–Nordström limit of the metric coefficient \tilde{A} in (39). Therefore, the junction condition becomes:

$$\partial_\eta \tilde{A}|_{\eta \rightarrow +0} - \partial_\eta \tilde{A}|_{\eta \rightarrow -0} = -16\pi \chi, \quad (46)$$

which yields the following relation between the Reissner–Nordström parameters and the dynamical LL -brane tension:

$$4\pi \chi r_0^2 + r_0 - m = 0, \quad \text{where } r_0 = m + \sqrt{m^2 - e^2}. \quad (47)$$

Eq. (47) indicates that the dynamical brane tension must be *negative*. Eq. (47) reduces to a cubic equation for the Reissner–Nordström mass m as function of $|\chi|$:

$$(16\pi |\chi| m - 1)(m^2 - e^2) + 16\pi^2 \chi^2 e^4 = 0. \quad (48)$$

In the special case of Schwarzschild wormhole ($e^2 = 0$) the Schwarzschild mass becomes:

$$m = \frac{1}{16\pi |\chi|}. \quad (49)$$

Notice that, for large values of the LL -brane tension $|\chi|$, m is very small. In particular, $m \ll M_{\text{Pl}}$ for $|\chi| > M_{\text{Pl}}^3$ (M_{Pl} being the Planck mass).

5. Conclusions

In the present Letter we have constructed a traversable wormhole solution by sewing together two copies of exterior Reissner–Nordström space–time regions at a Reissner–Nordström outer horizon via a *LL-brane* with a negative dynamical tension. This *LL-brane* provides a theoretically sound *non-phenomenological* gravitational source for the Reissner–Nordström wormhole since its dynamics, in particular its surface energy–momentum tensor, are derived from a well-defined world-volume Lagrangian action (1). Furthermore, let us stress that the *LL-brane* is electrically neutral and at the same time the Reissner–Nordström wormhole appears to possess two oppositely charged sources – one for each Reissner–Nordström region beyond the common horizon.

According to Eq. (47) (in particular Eq. (49)) wormholes built from *LL-branes* with very high negative tension have a small mass. To this end it is interesting to note that one can obtain baby universe solutions at very small energy cost by considering high surface tensions too, which similarly require a wormhole, although there the tension is positive (Refs. [15]). Notice however that in Refs. [15] the solutions are time-dependent and suffer from a singular initial problem. In the present work there is no singularity – the would-be singularities have been “surgically removed” by the wormhole matching.

On the other hand, for small values of the *LL-brane* tension $|\chi|$ Eq. (47) implies that the Reissner–Nordström geometry of the wormhole must be near extremal ($m^2 \simeq e^2$).

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